

# Superluminal Neutrinos in a Pseudoscalar Potential

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The superluminal propagation of neutrinos observed by OPERA collaboration can be interpreted as neutrinos traveling in a pseudoscalar potential which may be generated by a medium. The OPERA differential arrival time data set a constraint on the form of the pseudoscalar potential.

## I. INTRODUCTION

The OPERA collaboration recently reported the evidence of superluminal propagation of muon neutrinos [1]. An earlier report by the MINOS collaboration [2] reached the similar conclusion with a lower statistical confidence. If the result is not due to an unknown instrumental effect,

this fact inevitably points towards new physics. Since the release of the OPERA data, many ideas have been proposed to interpret the data, e.g. [3–8]. The difficulties of various interpretations have been discussed [9–13]. One of the difficulties is to account for the energy dependence of neutrino speed anomaly  $\Delta = (v - c)/c$ . While OPERA reports

$$\Delta = (2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.}) \times 10^{-5}, \bar{E}_\nu = 17 \text{ GeV} \quad (1)$$

$$\Delta_1 = (2.18 \pm 0.77(\text{stat.}) \pm 0.30(\text{sys.}) \times 10^{-5}, \bar{E}_{\nu,1} = 13.9 \text{ GeV} \quad (2)$$

$$\Delta_2 = (2.76 \pm 0.75(\text{stat.}) \pm 0.30(\text{sys.}) \times 10^{-5}, \bar{E}_{\nu,2} = 42.9 \text{ GeV} \quad (3)$$

for the anomaly in the entire sample,  $E_\nu < 20 \text{ GeV}$ , and  $E_\nu > 20 \text{ GeV}$  samples, respectively, SN 1987A neutrino arrival data poses a stringent constraint

$$|\Delta| < 2 \times 10^{-9}, \bar{E}_\nu = 10 \text{ MeV} . \quad (4)$$

The MINOS data, with less significance, suggests

$$\Delta = (5.1 \pm 2.9) \times 10^{-5}, \bar{E}_\nu = 3 \text{ GeV} . \quad (5)$$

This further sharpens the  $\Delta$  contrast in the 10 MeV to 10 GeV range.

Here we propose a superluminal neutrino theory without introducing Lorentz Invariance Violation (LIV) or particles with imaginary masses (tachyons). We envisage a pseudoscalar potential under the influence of which the neutrino propagates from CERN to OPERA detector.

## II. SUPERLUMINAL NEUTRINOS IN A PSEUDOSCALAR POTENTIAL

We introduce a pseudoscalar potential  $\phi > 0$ , which can be energy-dependent, but is constant in space (or can be approximated as a constant for the spatial range in consideration). The Dirac equation for a neutrino with mass  $m_\nu$  in such a pseudoscalar potential can be written as

$$[\alpha \cdot \mathbf{p} + \beta(m_\nu + \gamma_5 \phi)] \psi = E_\nu \psi, \quad (6)$$

where  $\alpha$ ,  $\beta$  and  $\gamma_5$  are the Dirac matrices. The natural units with  $c = 1$  and  $\hbar = 1$  are adopted here. The neutrino wave function  $\psi$  can be expressed in terms of two-component spinors  $u$  and  $v$  as

$$\psi = N \begin{pmatrix} u \\ v \end{pmatrix}, \quad (7)$$

where  $N$  is the normalization constant. These two spinors  $u$  and  $v$  satisfy the coupled equations

$$\begin{aligned} (\sigma \cdot \mathbf{p} - \phi) u - (E_\nu - m_\nu) v &= 0 \\ (\sigma \cdot \mathbf{p} + \phi) v - (E_\nu + m_\nu) u &= 0 \end{aligned} \quad (8)$$

Solving these two equations, we obtain the neutrino energy

$$E_\nu^2 = \mathbf{p}^2 + m_\nu^2 - \phi^2. \quad (9)$$

For  $m_\nu \ll \phi \ll E_\nu$ , one can write

$$|\mathbf{p}| \simeq E_\nu + \frac{\phi^2}{2E_\nu^2}. \quad (10)$$

The group velocity of the neutrino is then

$$v_g = \frac{dE_\nu}{d|\mathbf{p}|} = 1 + \frac{\phi^2}{2E_\nu^2} - \frac{\phi \dot{\phi}}{E_\nu}, \quad (11)$$

where  $\dot{\phi} = d\phi/dE_\nu$ , or

$$\Delta = v_g - 1 = \frac{\phi^2}{2E_\nu^2} - \frac{\phi \dot{\phi}}{E_\nu}. \quad (12)$$

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The condition of superluminal neutrinos ( $\Delta > 0$ ) can be written as

$$\frac{\phi}{E_\nu} > 2\dot{\phi}. \quad (13)$$

### III. CONSTRAINING THE FORM OF $\phi$ WITH DATA

The condition (13) is satisfied for a wide range of  $\phi$  forms. Below we attempt to construct an analytical form of  $\phi$  that gives superluminal motion and satisfies the observational constraints.

#### A. Constant $\phi$

An energy-independent potential ( $\dot{\phi} = 0$ ) naturally satisfies the condition (13), so that

$$\Delta = v_g - 1 = \frac{|\mathbf{p}|}{E_\nu} - 1 = \frac{\phi^2}{2E_\nu^2} > 1. \quad (14)$$

Such a form is similar to many theories invoking LIV or tachyon particles e.g. [8, 9, 13], which is ruled out by the OPERA differential speed data. The ratio of velocity difference in this model is  $\Delta_1/\Delta_2 = (\bar{E}_{\nu,2}/\bar{E}_{\nu,1})^2$ . According to equations (2) and (3), the ratio between two average neutrino energies is about  $\bar{E}_{\nu,2}/\bar{E}_{\nu,1} \sim 3.1$ . This corresponds to a predicted  $\Delta_1/\Delta_2 \sim 9.5$ , while the observed ratio is  $\Delta_1/\Delta_2 = 0.79^{+1.11}_{-0.50}$ . So a more complicated form of  $\phi$  with a much shallower  $E_\nu$  dependence on  $\Delta$  is needed.

#### B. Power law

For a pseudoscalar potential of the form

$$\phi(E_\nu) = AE_\nu^\alpha, \quad (15)$$

the superluminal condition (13) can be translated to

$$-0.5 < \alpha < 0.5. \quad (16)$$

The ratio of velocity difference in this model is  $\Delta_1/\Delta_2 = (\bar{E}_{\nu,1}/\bar{E}_{\nu,2})^{2\alpha-2}$ . Solving for  $\alpha$  using the OPERA data, one gets

$$0.72 < |\alpha| < 1.55 \quad (17)$$

with the typical value  $|\alpha| \sim 1.1$ . This range is inconsistent with the condition (16), and is in the subluminal regime. Therefore the power law model cannot interpret the OPERA data.

#### C. Power law exponential

The pseudoscalar potential of the form

$$\phi(E_\nu) = B \left( \frac{E_\nu}{E_0} \right)^\alpha e^{-E_\nu/E_0} \quad (18)$$

gives

$$\Delta = \frac{\phi^2}{E_\nu^2} \left( \frac{1}{2} - \alpha + \frac{E_\nu}{E_0} \right). \quad (19)$$

The superluminal condition is  $\alpha < 1/2 + E_\nu/E_0$ . However, when  $E_\nu \ll E_0$  and  $E_\nu \gg E_0$ , the velocity difference has the energy-dependence in the form of  $\Delta \propto E_\nu^{-2}$  and  $\Delta \propto E_\nu^{-1}$ , respectively. Both dependences are too steep to account for the OPERA data in equations (2) and (3). This form is therefore not favored.

#### D. Power law logarithmic

We consider the potential of the form

$$\phi(E_\nu) = C \left( \frac{E_\nu}{E_0} \right)^\alpha \ln \left( \frac{E_\nu}{E_0} \right). \quad (20)$$

The superluminal condition (13) can be translated to

$$\frac{E_\nu}{E_0} > \exp \left( \frac{2}{1-2\alpha} \right) \quad (21)$$

for  $\alpha < 1/2$  (the branch  $\alpha > 1/2$  is not favored since the  $E_\nu$ -dependence of  $\Delta$  is very steep). The velocity difference takes the form

$$\Delta = \frac{C^2}{E_\nu^2} \left( \frac{E_\nu}{E_0} \right)^{2\alpha} \ln \left( \frac{E_\nu}{E_0} \right) \left[ \left( \frac{1}{2} - \alpha \right) \ln \left( \frac{E_\nu}{E_0} \right) - 1 \right]. \quad (22)$$

For a not very high  $E_0$ , the superluminal condition (21) is easily satisfied. The steep  $\propto E_\nu^{-2}$  dependence is compensated by other factors in equation (22) so that a shallow  $E_\nu$ -dependence can be achieved in the superluminal regime. By properly adjusting  $E_0$  and normalization  $C$ , the OPERA data can be interpreted.

### IV. DISCUSSION

We have shown that if neutrinos travel in a pseudoscalar potential, superluminal propagation is possible under the condition given in equation (13), without violating special relativity. In order to interpret the OPERA data, a shallow energy-dependence is required in the superluminal regime. A potential form similar to equation (20) with  $\alpha < 1/2$  can meet such a requirement.

The very small  $|\Delta|$  derived for SN 1987A of 10 MeV neutrinos is difficult to account for with such a potential. If one adjusts  $E_0$  to interpret OPERA data, the 10 MeV

neutrinos would be in the subluminal regime with a large  $|\Delta|$  violating the data constraint. If one instead adjusts 10 MeV to the transition point from superluminal to subluminal, i.e.  $E_\nu \sim E_{\nu,c} = E_0 \exp(\frac{2}{1-2\alpha}) \sim 10$  MeV, then the energy-dependence in the 10 GeV range is too steep to satisfy the OPERA data. In order to incorporate the SN 1987A data, one needs to either argue for a local effect of OPERA anomaly (e.g. the pseudoscalar potential is related to the density, gravity or magnetic fields in the neutrino propagation path in the earth crust) or admit that the potential (20) is not the correct form in the low energy regime.

If, however, the pseudoscalar potential (20) can be extended to the high energy regime, it is interesting to note that in the  $\sim$  PeV energy range where the internal-shock-origin neutrinos from gamma-ray bursts (GRBs) are supposed to be generated [14], neutrinos are still superluminal but with a much smaller  $\Delta \sim 10^{-10}$ . For typical high-

luminosity GRBs at cosmological distances, the lead time would be several years. For nearby low-luminosity GRBs [15, 16], the lead time can be as short as several months. This can be in principle tested by independent discoveries of a high-energy neutrino burst detected by IceCube and a later GRB detected in the same direction. This may be possible for a bright, nearby GRB event such as GRB 030329. The TeV neutrinos [17, 18] have a much larger  $\Delta \sim 10^{-8}$ , and hence, a leading time of decades to centuries, which is difficult to test observationally.

Finally, we'd like to comment on that the normalization parameter  $C$  in (20) adjusted to fit the OPERA data naturally leads to  $\phi \gg m_\nu$ , but  $\phi \ll m_e$ . So it is allowed that the Dirac equation with pseudoscalar potential (6) also applies to electrons and other spin 1/2 leptons, but those particles cannot be superluminal since their masses dominate the  $\phi$  term in the energy-momentum equation (9).

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